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23 June 1997



Evan Kwerel
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Dear Evan,

After our conversation last week, I relayed to Frank Kelly your suggestion that he drop by your office when he is in D.C. for the International Teletraffic Congress at the Marriott, which runs June 22-27. (Frank is giving a Plenary Talk, entitled 'Tariffs and Traffic', on Thursday at 9.00 am.)

I am enclosing the latest revision of our paper, 'A Combinatorial Auction with Multiple Winners for COLR' (9 June 1997). In this revision, we explicitly address the issue of minimising the possibility of bidder collusion (see pages 4 and 6). We achieve this by conducting Stage 1 as a sealed-bid auction, with each bidder submitting a panel of bids on individual properties (similar to the Vincent proposal). Stage 2 (combinatorial bidding) is as before, although we now recommend three substages (rather than two), so this stage now progresses along the lines of the PCS auctions.

I hope you and Frank have the opportunity to meet. In any case, please let me know if I can provide any clarification or elaboration on our paper.

Yours sincerely,

**Richard Steinberg**

enclosure

# A Combinatorial Auction with Multiple Winners for COLR

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University of Cambridge

Revised: 9 June 1997

## Executive Summary

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR (Carrier of Last Resort) responsibility. The auction incorporates synergies by permitting all combinatorial bids, allows for and determines the number of multiple winners, and minimises the possibility of bidder collusion. In addition, the procedure is computationally simple for the auctioneer and thus is very efficient to run.

The inherent computational complexity of combinatorial bidding cannot be eliminated. However, the computational burden of evaluating synergies rests with the bidders claiming those synergies, while the auctioneer simply checks that a bid is valid. There is very little computational burden for small players interested in only a small number of assets. If *no* synergies are claimed, then the auction reduces to a simple sealed-bid auction.

## 1. Overview of the Auction

Define a **PAUSE (Progressive Adaptive User Selection Environment) Auction** to be a two-stage auction with no bid withdrawals and no bid waivers, where:

(i) Stage 1 is a simultaneous, sealed-bid auction, with bidders submitting a panel of bids on each individual property to facilitate multiple winners; and

(ii) Stage 2 is a simultaneous, multiple-round auction, conducted in three substages, with progressive eligibility requirements and an exact improvement margin requirement, with combinatorial bids submitted via an AUSM (Adaptive User Selection Mechanism[1]), to facilitate realisation of player synergies.

The auction is also designed to minimise the possibility of collusion. This is detailed in Section 3.

The PAUSE auction is designed to be fully general in that every possible combinatorial bid is available to the bidders. If, however, the auctioneer wishes to restrict the bids in any manner that he finds convenient to verify, the auction structure will accommodate this, and the auctioneer can announce to the bidders a list of attributes a bid must have. (An example of such an attribute might be: 'bids that are combinatorial are to be composed of geographically contiguous subsets of the properties'.) This is formalised in the next section.

## 2. Definitions

Label *properties* $j \in J$, and *blocks* $k \in K$, where $K = K(J, A)$ is a set of subsets of $J$ defined by a set of *attributes* $A$ that are computationally simple for the auctioneer to verify for each member of $K$. Let

$$K_n = \{k \in K(J, A) : 1 \leq k \leq n\},$$

where $|k|$ is the number of properties in block $k$.

(Thus, $K_1$ is the set of blocks allowed by the attribute set and consisting of a single property, $K_2$ is the set of allowed blocks consisting of at most two properties, $K_3$ is the set of allowed blocks consisting of at most three properties, and so forth.)

A *partition* $P = (p_1, p_2, \ldots, p_r)$ is a collection $p_1, p_2, \ldots, p_r \in K$ such that $\bigcup_{i=1}^{r} p_i = J$, and $p_i \cap p_j = \varnothing$, $i \neq j$.

[1] Bykowsky, M.M., R.J. Cull, and J.O. Ledyard, 'Mutually Descriptive Bidding: The FCC Auction Design Problem', *Social Science Working Paper* 916, California Institute of Technology, 1995.

(In words, a partition is a grouping of all the properties in the auction into sets that do not overlap.)

A *composite bid* comprises a partition $P = (p_1, p_2, ..., p_r)$ together with an *evaluation*

$$(C(P); c(p_1), c(p_2)...., c(p_r))$$

where

$$C(P) = \sum_{i=1}^{r} c(p_i), \qquad (*)$$

and $c(p_i)$ is the *bid* for block $p_i$.

To be more precise, $c(p_i)$ is *the value of the bid for block* $p_i$. A composite bid consists of $3r+1$ pieces of information, capable of registration in a database. The first piece of information is the total value of the composite bid, *C(P)*. The $3r$ pieces of information are, for each $i$ ($i = 1, 2, ..., r$): (1) the specification of the block $p_i$, (2) the value of the bid on the block, $c(p_i)$, and (3) the identity of the bidder for block $p_i$.

Note that $c(p_i)$ is the *total subsidy for block* $p_i$. It corresponds to a *subsidy per subscriber in block* $p_i$ of $c(p_i)/\|p_i\|$, where $\|p_i\|$ is the total number of subscribers in all the properties in $p_i$.

Items (1) and (2) are available from the database to all bidders; item (3) may be available only to the auctioneer and the bidder concerned, or may be public information.

## 3. The Procedure

## Opening Bids

This analysis does not attempt to determine the merits of historical versus forward-looking cost models. However, opening bids for each property could be the lower of the historical cost and the forward-looking cost for that property. (By 'forward-looking cost' we mean, for example, the Total Service Long Run Incremental Cost or the cost obtained from the Benchmark Cost Model.)

If the lower of these two costs is the historical cost, then it is announced that historically service has been provided on this property at a certain subsidy level and it is expected that service will be provided at no higher than that level in the future.

If, on the other hand, the lower of these two costs is the forward-looking cost, then forward-looking cost would serve as a starting point for our analysis to determine the minimum subsidy to provide service in a given market.

## Stage 1 - Bidding on Individual Properties.

**The Bidders:** Each bidder submits a panel of bids for each individual property on which he has an interest. The panel of bids from player $i$ on a given property is (c(1,i), c(2,i), ..., c(M,i)). Here, c(m,i) is the subsidy player $i$ requires to be one of m multiple winners on the property. (We would expect the sequence {c(m,i)} to be increasing in m.)

**The Auctioneer:** The auctioneer prefers more multiple winners to less. Let f(m) be the fractional cost saving the auctioneer is prepared to forgo to have m winners on a property rather than a single winner. Here f(m) can be described as the *m-competitor discount factor*. (Here, f(1) = 0, and we would expect the function f(m) to be increasing in m.)

**Multiple Winners:** After all the sealed bids have been submitted, the auctioneer computes, for each property, C(m) = the sum of the smallest m terms from the collection {c(m,i), i = 1, 2, .., I}. The smallest m terms identify the least costly way to subsidy the property with m multiple winners.

After computing, for each property, C(m) for each value of m, the auctioneer next computes, for each property, the value m* as the argument that minimises (1 - f(m))·C(m). The auctioneer announces m* as the number of multiple winners for that property, and the bids achieving the minimum C(m*) as the m* winning bids at the end of Stage 1.

Before the start of Stage 2, property $j$ is replaced by *m(j)* properties $j_1, j_2, j_3, \ldots, j_{m(j)}$, each with a nominal number of subscribers equal to *sub(j)* / m(*j*), where *sub(j)* denotes the number of subscribers in property *j*.

**Minimisation of Bidder Collusion:** Collusive behaviour known as signalling is impossible in a sealed-bid, single-round auction. However, such an auction also allows for no transmittal of information to the players about the aggregate interest--i.e., the market--for each of the properties, and thus may inhibit players to commit to bid without learning more about the general state of the market, especially with regard to properties that are valuable to them only in combinations. Thus, this auction is conducted in two stages, where the second stage is conducted as a simultaneous, multiple-round auction with combinatorial bids.

## Stage 2 - Combinatorial Bidding

**The Bidders:** Each bidder submits a single composite bid on a collection of properties, where each bidder's partition $P = (p_1, p_2, \ldots, p_r)$ is restricted to $p_i \in K_n$, where $c(p_i)$ is either a new bid for block $i$, or a registered bid. Initially, $n = 2$. For a composite bid to be valid, for each property $j$ the bid must not allocate $j_s$ and $j_t$ $(s \neq t)$ to the same player. In this stage of the auction, the bidder identities are to made public.[2 3] Thus, the validity of a composite bid--and in particular the requirement that the bid does not allocate $j_s$ and $j_t$ $(s \neq t)$ to the same player--can be checked by the player constructing the composite bid.

**The Auctioneer:** In each round, the auctioneer checks that a composite bid is *valid* by checking:

(i) Bid Validity: each bid claiming to be registered is indeed registered in the database; that new bids satisfy $p_i \in K_n$, that is, that new bids are on allowed blocks of not more than $n$ properties; and, for each property $j$, the composite bid does not allocate $j_s$ and $j_t$ $(s \neq t)$ to the same player

(ii) Evaluation Validity: equation (*) holds, i.e., the value $C(P)$ of the composite bid is indeed the sum of the bids on each of its blocks, and

(iii) Increment Validity: bid $C(P)$ is less than the last accepted bid by *exactly* the specified bid increment.

In each round of Stage 2, the new collection of bids on the blocks $\{c(p_i)\}$ are registered to their respective owners, and the lowest valid composite bid is accepted. The round ends when bidding ends. Stage 2 is divided into three substages.

**Activity Rules:** A bidder is *active* on a property if his bid on a block containing that property forms part of the accepted composite bid of the previous round, or if he submits a valid bid in the current round on a block containing that property. Each of the three substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering 60 per cent in Stage 1, 80 per cent in Stage 2, and 95 per cent in Stage 3, of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to substage 2 occurs when there are bids on no more than 10 per cent of the total number of

[2] Note that synergies are accounted for via composite bids. Thus, to allow for multiple winners, players need to check the validity of their composite bids, which is not possible with a sealed-bid auction.

[3] R.P. McAfee and J. McMillan ('Analyzing the Airwaves Auction', Journal of Economic Perspectives 10, 1996) report that in the MTA broadband PCS auction, the FCC revealed bidders' identities, judging that the risk of collusion was outweighed by the benefits of information. As McAfee and McMillan point out: 'Bidders' identities are useful to the bidders for evaluating the meaning of others' bids, reducing the winner's curse and generally assisting sensible bidding.' They add that '[i]t takes only one maverick bidder to upset an attempt at collusion', and provide an illustrative example from the MTA auction. With synergies, one might expect the overlapping nature of composite bids would tend to make collusion all the more difficult.

subscribers for three consecutive rounds; the transition from substage 2 to 3 occurs when there are bids on no more than 5 percent of the subscribers for three consecutive rounds. The auction authority may wish to retain discretion to modify these numbers during the auction.

**Bid Increments:** In each round there is an *exact improvement margin requirement*:

If $c(p_1), c(p_2) \ldots, c(p_s)$ are the *new* bids in a composite bid, then the evaluation must improve on the previous best evaluation by *exactly* $\varepsilon s$, i.e., an improvement of $\varepsilon$ per block on average.

**Multiple Winners:** At the conclusion of Stage 2, the $m(j)$ winners on property $j$ are each designated a $1/m(j)$ share of the responsibility on property $j$. Specifically, the contractual obligation carried by each player is as follows: *The player will receive his bid subsidy per subscriber on* up to $1/m(j)$ *of the total number of subscribers in that property, and he is required to serve* at least $1/m(j)$ *of the subscribers in that property.* The particular subscribers that make up this fraction are not specified; the player will *compete* for these subscribers with the other winners on that property. If a subscriber is unserved in a property with multiple winners, the regulatory authority may require any one of the multiple winners who is not serving the full amount of his contractual share to serve that subscriber. (There is thus a considerable incentive for players to actively seek to serve their share of subscribers, lest they be required to serve subscribers not of their choice.)

A player's winning bid on property $j$ will, in general, be part of a composite bid. Thus, the limitation on the fraction of customers for which a player will be subsidised prevents the player from cross-subsidising property $j$ from the other properties that comprise its bid. Of course, each player is free to compete for any or all the customers in property $j$, although it will not receive subsidy for any customers beyond the fraction it has won in the auction.

**Minimisation of Bidder Collusion:** The opportunity for two types of bidder collusion is minimised, each of which is a form of signalling. *Property-preference signalling* is where bidders indicate to each other their preferences for particular properties, with the intention of bidding noncompetitively on properties of strong interest to other bidders but not themselves. This type of explicit collusion is minimised via the exact improvement margin requirement. This approach is consistent with the rounding idea originally considered for the PCS auctions, but has other advantages, including the minimisation of the computational burden on the bidders. *Price-level signalling* is where two or more bidders realise their common preference for a particular property they can share as winners and, consequently, on which they bid non-competitively. This type of implicit collusion is minimised in Stage 2 by having the number of multiple winners determined in Stage 1; thus, most of the bidder surplus for the individual properties has already been extracted. Any additional surplus is most likely to be due to synergies, for which price-level signalling is both very difficult, and, very difficult to take advantage of even if successful, due to the very nature of combinatorial bids.

## 4. Other Auction Rules

### Bid Withdrawals

No bid withdrawals are allowed in either stage.

In the PCS auctions, bid withdrawals were permitted. Specifically, a high bidder withdrawing his bid during the course of the auction was required to pay the difference between his bid and the price for which the licence ultimately sold; a winning bidder withdrawing after the close of the auction suffered an extra penalty. It may be asked why bid withdrawals were permitted, since they complicate the auction. Paul Milgrom, in his attachment to GTE's Comments[4], clearly states the motivation: 'In effect, a bid withdrawal substitutes partially and quite imperfectly for combinatorial bidding.'

### Bid Waivers

For simplicity, there are no bid waivers in either stage.

[4] Statement of Paul R. Milgrom Attached to GTE's Comments in Response to Questions, CC Docket No. 96-45.

## 5. Discussion

### Bid Increment and Block Size

McAfee and McMillan (1996)[5] report that in the MTA auction[6] (in which the highest bid won the licence), aggressive bidding in early rounds took the form of 'jump bidding': entering bids far above the required minimum bid increment. Analogously, jump bidding in the COLR market would mean entering bids far below the required minimum bid increment. In a combinatorial auction, jump bidding for a block of several properties would be effective at preventing small players from piecing together a comparable composite bid (the threshold effect). The rule that the improvement margin must be an exact increment is designed to lessen the threshold effect. It also helps keep the computation requirement down, by limiting the ranges of possibilities that need to be considered by bidders.

The size of the bid increment $\varepsilon$ and the rate of increase of the block size limit, $n$, are used by the auctioneer to control the speed of the auction in conjunction with the activity rules. For example, the auctioneer might move $n$ from the starting value of 2, to 3, 4, 5, ... ; however the auctioneer might instead move $n$ to 4, 8, 16, ... . In either case the value of the bid increment would decrease, and the activity rule percentage increase, as $n$ increases.

### Multiple Winners

As an example, suppose that for a property $j$ the number of multiple winners determined in Stage 1 is $m(j) = 3$. Then property $j$ is replaced by three 'properties' $j_1$, $j_2$, and $j_3$. Throughout Stage 2, all composite bids on these three 'properties' are required to be composed of bids from precisely 3 players. At the conclusion of Stage 2, there will be 3 winners on the property, each with 1/3 share.

The winning bid of each firm may in general be part of a block that includes other properties as well as property $j$. The winning bid on the block is the total subsidy that the firm will receive collectively for all customers in all properties contained in that block. The total subsidy for the block divided by the number of subscribers in all the properties in that block yields the subsidy per subscriber over that block. Let the subsidy per subscriber for the three winning firms over their respective blocks that include property $j$ be $S_1$, $S_2$, $S_3$. Here $S_1$, $S_2$ and $S_3$ may differ. It should be emphasised that the subsidy per subscriber is defined over a block, and is conditional on the player winning the bid--and thus receiving the subsidy--collectively over all the properties of the block.

The contractual obligation carried by each player on property $j$ is as follows: Player i ($i = 1, 2, 3$) will receive a subsidy of $S_i$ on up to 1/3 of the subscribers in property $j$ and is required to serve up to 1/3 of the subscribers in property $j$.

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[5] Reference provided in footnote 3.

[6] The MTA auction ran from December 1994 to March 1995 and sold broadband licenses covering the 51 'Major Trading Areas', or MTAs, into which the United States is divided.

Finally, note that it is essential that, before the start of Stage 2, the auctioneer specifies the rules that need to be satisfied by a valid composite bid in a manner that can be checked by players, as well as by the auctioneer. Further, the auctioneer should not attempt to decide the number of multiple winners after the conclusion of the auction, since to do so would involve the auctioneer in a task of some considerable computational complexity.

## Contractual Obligation and Price

If a fixed number of multiple winners will be accepted on a given property, and the contracts for each will carry the same contractual obligation, then rational behaviour by the bidders will generally lead to them achieving the same price (within $\varepsilon$) on successful bids on blocks comprising just that property. This is simply the law of one price, i.e., a bidder is unlikely to pay more for something identical available at a lower price. Of course the bounded rationality of players, together with the inherent computational complexity of combinatorial bidding, may cause bidders to occasionally depart from the law of one price. Note also that a price for a property cannot be determined from a composite bid if within that composite bid the property is part of a larger block. Similarly, if the contracts carry different obligations, then rational behaviour by the bidders will lead to a variety of achieved prices reflecting the bidders' views about the value to the bidders of the various obligations.

## Costs Studies and Administrative Costs

Note that no new cost studies are required for participation in this auction. To bid, each firm needs to know only the value of its synergies, something that such a firm most likely already calculates and is part of its information.

The costs of auction administration will be minimal. Bounds on the number of rounds, and on the maximum number of bids that need to be registered, are provided in the Technical Appendix.

# Technical Appendix: Computational Complexity of the Auction

## Number of Rounds

Since in each round of Stage 2 the value of the accepted composite bid must decrease by at least $\varepsilon$ over the previously accepted composite bid, the number of rounds in total is bounded above by $C_0(P_0)/\varepsilon$, where $C_0(P_0)$ is the value of the opening composite bid (perhaps set by the auctioneer).

## Number of Registered Bids

Let $B$ be the number of bidders. Since each bidder is allowed to make at most one composite bid per round, the maximum number of bids that needs to be registered by the auctioneer is bounded above by

$$\frac{C_0(P_0)}{\varepsilon} B|J|.$$

## Discussion

In general, it may be an NP-complete problem for a bidder to determine whether he can make a composite bid that beats the currently accepted composite bid. The results of Rothkopf *et al.* (1996)[7] show that, if the form of composite bids is restricted in one or other of several possible ways, then the problem becomes manageable. However bidders are unlikely to agree upon the form of the appropriate restriction on composite bids. We view the elicitation of the form and size of potential synergies as a major purpose of the auction.

Work on computationally difficult problems shows that in several situations where finding the exact optimum is hard, finding a good approximation to the optimum with high probability may be relatively easy (Jerrum and Sinclair 1996)[8]. It is our belief that the traditional problems of elicitation and gaming are more serious difficulties than the possible computational burden on those bidders claiming complex synergies.

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[7] Rothkopf, M.H., A. Pekec, and R.M. Harstad, 'Computationally Manageable Combinational Auctions', RUTCOR, Rutgers University, May 1996.

[8] Jerrum, M. and A. Sinclair, 'The Markov Chain Monte Carlo Method: An Approach to approximate Counting and Integration', in Dorit S. Hochbaum (ed.), *Approximation Algorithms for NP-Hard Problems*, PWS Publishing Company, Boston, Massachusetts, 1996.